Student No.:

COLLEGE OF SCIENCE AND TECHNOLOGY
ROYAL UNIVERSITY OF BHUTAN
PHUENTSHOLING: BHUTAN

SPRING SEMESTER EXAMINATION: 2018

Class	**:**	**B.E. First Year IT**
Module	**:**	**Discrete Mathematics**
Module Code	**:**	**MAT110**
Max. Marks	**:**	**70**
Max. Time	**:**	**3 Hrs.**

General Instructions:

1. ***All the questions are compulsory from Question No. 1, Question No. 2 and Question No. 3.***
2. ***Answer any three questions from Question No. 4.***
3. ***Sketch the figures neatly wherever necessary.***
4. ***Usage of pencil apart from figures will be treated as rough work.***
5. ***Do NOT write anything on this question paper.***

Question No. 1 **[$10 \times 2 = 20$]**

1.1 Show the following tautological implication using truth table.

$$\sim (P \to Q) \Rightarrow \sim Q$$

1.2 Write each of the following statements in the form "if P, then Q" in English.
(a) It is necessary to wash boss's car to get promoted.
(b) Jigme will go swimming unless the water is too cold.

1.3 Express the following statement using quantifiers and predicates. Obtain its negation and write in simple English.
"There is someone in the class who has a good attitude."

1.4 Consider the binary operation $*$ on a set of real numbers $\mathbb{R}$, where $a * b = a^b$; $\forall a, b \in \mathbb{R}$. Determine whether $(\mathbb{R}, *)$ is associative or not.

1.5 Define a binary tree and a full binary tree. Provide an example each.

1.6 Is there a graph with the degree sequence (1, 1, 3, 3, 3, 4, 6, 7)? If so, draw a graph.

1.7 Find the sets A and B if $B - A = \{8, 11\}$, $A - B = \{1, 2, 6, 9\}$ and $A \cap B = \{4, 5\}$. What are the cardinalities of set A and B?

1.8 In a survey of 70 students, it was observed that 30 students study Discrete Mathematics, 25 study Java programming, 28 study C++ programming, 15 study both Discrete

Mathematics and C++ programming, 18 study Java programming and C++ programming, 20 study Discrete Mathematics and Java programming, and 5 study all the three subjects. Find

(a) The number of students who study at least one of the three subjects.

(b) The number of students who do not study any of the three subjects.

1.9 Use the method of iteration to find the explicit formula for the recurrence relation

$$a_n = 2a_{n-1};\ a_0 = 1.$$

1.10 Find the particular solution to the recurrence relation

$$a_n - 6a_{n-1} + 9a_{n-2} = 7.$$

Question No. 2 **[$4 \times 3 = 12$]**

2.1 Prove that

$$(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$$

without constructing truth table.

2.2 Examine the graphs given in **Fig.2.2** for isomorphism. Justify your answer.

2.3 Show that $(ab)^{-1} = b^{-1} \cdot a^{-1}$ for all $a, b \in G$.

2.4 Let the set $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Compute the product

$$p = (2\ 6) \circ (3\ 5\ 7\ 8) \circ (2\ 5\ 3\ 4).$$

Determine whether the permutation p is an even or odd.

Question No. 3 **[$5 \times 4 = 20$]**

3.1 Eight towns labelled a to h, are to be connected by a telephone network. The network cable costs Nu. 15,000 per kilometre to lay. The required connections and distances (in km.) between the towns for which we require a connection are given in the **Fig.3.1**. Use Kruskal's algorithm to find the network of minimum cost. What is the minimum cost?

3.2 Consider the group $G = \{1, 2, 4, 7, 8, 11, 13, 14\}$ under multiplication modulo 15.

(a) Construct the multiplication table of G.

(b) Find $4^{-1}, 7^{-1}$, and 11^{-1}.

(c) Find the orders and subgroups generated by $2, 7$, and 11.

3.3 Find the number of different paths of length 3 from vertex e to c in the undirected graph shown in **Fig.3.3**. Identify those paths from the graph.

3.4 Show the following argument form

$R \vee S, \sim Q \to \sim U, \sim (P \vee \sim Q) \to U, \sim S \to \sim P, \sim S, \therefore Q$

is a valid by deducing the conclusion from the premises step by step through the use of basic inference rules or laws of logic.

3.5 Find the total solution of the recurrence relation

$$a_{n+2} - 5a_{n+1} + 6a_n = n^2$$

using the characteristic root method.

Question No. 4 (Answer any three questions) [$3 \times 6 = 18$]

4.1 Use Dijkstra's algorithm to find the shortest path between the vertices a and z from the weighted graph in **Fig.4.1**. Identify that path from the graph.

4.2 Let $S = \mathbb{Q} \times \mathbb{Q}$, the set of ordered pairs of rational numbers, where $*$ is a binary operation befined by

$$(a, b) * (c, d) = (ac, ad + b).$$

(a) Prove that $(S, *)$ is a semi-group.

(b) Find the identity element and inverse element of S if it exists.

4.3 Use the Principle of Mathematical Induction to show that

$$\frac{1^2}{1 \cdot 3} + \frac{2^2}{3 \cdot 5} + \cdots + \frac{n^2}{(2n-1) \cdot (2n+1)} = \frac{n(n+1)}{2(2n+1)}, \quad \text{for } n \geq 1.$$

4.4 Prove that $\forall x[P(x) \rightarrow \sim Q(x)]$ is a valid inference from the premises $\exists x[P(x) \wedge Q(x)] \rightarrow \forall y[R(y) \rightarrow S(y)]$ and $\exists y[R(y) \wedge \sim S(y)]$.

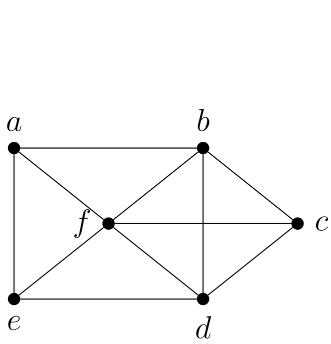

Figure 2.2

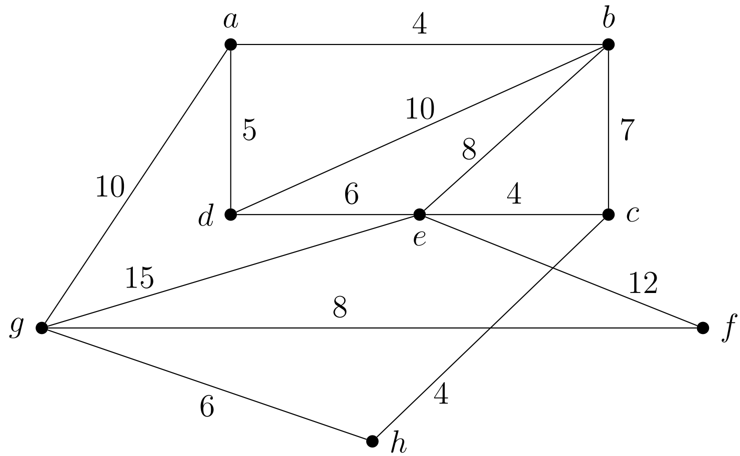

Figure 3.1

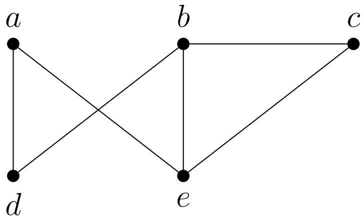

Figure 3.3

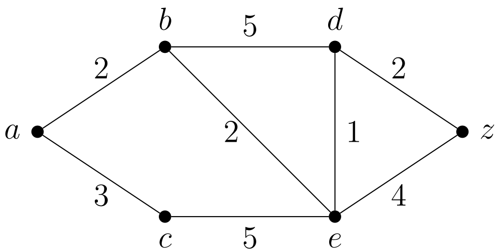

Figure 4.1